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Explicit solutions for some (2 + 1)-dimensional nonlinear evolution equations

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Abstract

Some (2 + 1)-dimensional nonlinear evolution equations, including the Kadomtsev–Petviashvili II (KPII) equation and the modified Kadomtsev–Petviashvili (KP) equation, are decomposed into two (1+1)-dimensional soliton systems in the coupled KdV hierarchy. With the help of the decomposition and the Darboux transformation, some explicit solutions of these (2+1)-dimensional nonlinear evolution equations such as new soliton solutions of the KPII equation and the modified KP equation are obtained.

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(Some figures in this article are in colour only in the electronic version)

The Kadomtsev–Petviashvili (KP) equation [1] is a ubiquitous nonlinear wave equation governing weakly nonlinear long waves in two dimensions with slow transverse variations and has also been proposed as a model for surface waves and internal waves in straits or channels of varying depth and width [1–4]. The KP equation is also the two-dimensional generalization of the KdV equation and was widely investigated. Most of the information about its solutions may be extracted from the Lax pair. Various systematic methods [5–14] have been developed to obtain exact solutions of the KP equation such as the inverse scattering transformation, the bilinear transformation of Hirota, the dressing method, the Bäcklund and Darboux transformations, the algebraic curve method and the nonlinearization approach of eigenvalue problems [15, 16]. Some important explicit solutions are found, including the N-soliton solution, the quasi-periodic solution, the rational solution and others.

In this paper, we shall study explicit solutions of (2 + 1)-dimensional nonlinear evolution equations

$$q_t = \tfrac{1}{4}\left(q_{xxx} + 12qq_x + 3\partial_x^{-1}q_{yy}\right) \tag{1}$$

$$q_t = q_{xxx} + 6qq_x - \tfrac{3}{4}\left[q^{-1}q_x^2 - q^{-1}\left(\partial_x^{-1}q_y\right)^2\right]_x \tag{2}$$

and

$$r_t = \tfrac{1}{4}\left(r_{xxx} - 6r^2 r_x + 6r_x\partial_x^{-1} r_y + 3\partial_x^{-1} r_{yy}\right) \tag{3}$$

$$r_t = r_{xxx} - 6r^2 r_x + 3rr_y + 3r_x\partial_x^{-1} r_y \tag{4}$$

$$r_t = \tfrac{1}{2}\left(-r_{xxx} + 6r^2 r_x - 6rr_y + 3\partial_x^{-1} r_{yy}\right) \tag{5}$$

$$r_t = -rr_y + r_x\partial_x^{-1} r_y + \partial_x^{-1} r_{yy} \tag{6}$$

$$r_t = \tfrac{1}{2}\left(r_{xxx} - 6r^2 r_x + 2rr_y + 4r_x\partial_x^{-1} r_y + \partial_x^{-1} r_{yy}\right) \tag{7}$$

where ∂_x^{-1} represents an inverse operator of $\partial_x = \partial/\partial x$ with the condition $\partial_x\partial_x^{-1} = \partial_x^{-1}\partial_x = 1$, which can be defined as $(\partial_x^{-1} f)(x) = \int_{-\infty}^{x} f(x')\,dx'$ under the decaying condition at infinity. Equations (1) and (3) are the famous Kadomtsev–Petviashvili II (KPII) equation and the mKP equation [17]. Here we shall give their new soliton solutions. It is very interesting that all the equations (3)–(7) are representable in a unified way

$$r_t = \frac{1}{\alpha+\beta}\left[\left(\alpha - \tfrac{1}{2}\beta\right) r_{xxx} - 6\left(\alpha - \tfrac{1}{2}\beta\right) r^2 r_x + 3(\alpha-\beta) rr_y + 3\alpha r_x\partial_x^{-1} r_y + \tfrac{3}{2}\beta\partial_x^{-1} r_{yy}\right] \tag{8}$$

with constants α and β, which is a generalized (2 + 1)-dimensional mKdV equation. In fact, equation (8) is respectively reduced to (3)–(7) for $\alpha = \beta$, $\beta = 0$, $\alpha = 0$, $\alpha = \frac{1}{2}\beta$ and $\alpha = 2\beta$.

In what follows we outline the methods used here and give the main results. To decompose the (2+1)-dimensional equations (1)–(7) into (1+1)-dimensional soliton equations, we consider the first two systems of the coupled KdV hierarchy [18]

$$\begin{aligned} u_y &= u_{xx} + 2uu_x + 2v_x \\ v_y &= -v_{xx} + 2(uv)_x \end{aligned} \tag{9}$$

and

$$\begin{aligned} u_t &= u_{xxx} + \left(6uv + 3uu_x + u^3\right)_x \\ v_t &= v_{xxx} + \left(-3uv_x + 3v^2 + 3u^2 v\right)_x. \end{aligned} \tag{10}$$

It is well known that equations (9) and (10) are compatible since the flows determined by them commute. We now assume that (u, v) is a compatible solution of (9) and (10), and introduce two functions q and r by

$$q(x, y, t) = v(x, y, t) \tag{11}$$

$$r(x, y, t) = u(x, y, t). \tag{12}$$

From (9) and (10) we have

$$\begin{aligned} &\partial_x^{-1} q_y = -v_x + 2uv \\ &\tfrac{3}{4}\partial_x^{-1} q_{yy} - \tfrac{3}{4} q_{xxx} - 3qq_x = -3(uv_x)_x + 6uvu_x + 3u^2 v_x \\ &q_t - q_{xxx} - 6qq_x = -3(uv_x)_x + 6uvu_x + 3u^2 v_x \end{aligned} \tag{13}$$

which implies the KP equation (1). Equation (9) can be written as

$$u = \tfrac{1}{2} q^{-1} q_x + \tfrac{1}{2} q^{-1}\partial_x^{-1} q_y \tag{14}$$

$$v = -\tfrac{1}{2}\left(r_x + r^2 - \partial_x^{-1} r_y\right). \tag{15}$$

Substituting (14) into the first expression of (10) yields (2). Equations (3)–(7) can be obtained through elementary calculations. Therefore, we have the following assertion.

Proposition 1. *Let (u, v) be a compatible solution of the (1+1)-dimensional soliton systems (9) and (10). Then the function $q(x, y, t)$ determined by (11) solves the (2 + 1)-dimensional equations (1) and (2), the function $r(x, y, t)$ by (12) solves any one of the (2 + 1)-dimensional equations (3)–(7).*

Noticing (11), equation (15) reads

$$q = -\tfrac{1}{2}\big(r_x + r^2 - \partial_x^{-1} r_y\big) \tag{16}$$

by which a direct calculation shows that

$$\begin{aligned} &q_t - \tfrac{1}{4}\big(q_{xxx} + 12qq_x + 3\partial_x^{-1} q_{yy}\big) \\ &\quad = -\tfrac{1}{2}\big(\partial_x + 2r - \partial_x^{-1}\partial_y\big)\big[r_t - \tfrac{1}{4}\big(r_{xxx} - 6r^2 r_x + 6r_x\partial_x^{-1} r_y + 3\partial_x^{-1} r_{yy}\big)\big]. \end{aligned} \tag{17}$$

This means that (16) is the Miura transformation between the KPII and mKP equations [17]. Here we obtain the Miura transformation in a simple way. In the following, we shall construct a Darboux transformation (DT) [8, 19–22] of (9) and (10). With the help of theorem 1, we give explicit solutions of the (2 + 1)-dimensional equations (1)–(7).

Equation (9) has the Lax representation [23]

$$\phi_x = U(u, v, \lambda)\phi \qquad U(u, v, \lambda) = \begin{pmatrix} -\frac{1}{2}\lambda + \frac{1}{2}u & -v \\ 1 & \frac{1}{2}\lambda - \frac{1}{2}u \end{pmatrix} \tag{18}$$

$$\phi_y = V^{(1)}(u, v, \lambda)\phi \qquad V^{(1)}(u, v, \lambda) = \begin{pmatrix} \frac{1}{2}(u_x + u^2 - \lambda^2) & v_x - uv - \lambda v \\ \lambda + u & -\frac{1}{2}(u_x + u^2 - \lambda^2) \end{pmatrix} \tag{19}$$

where u and v are two potentials and λ is a constant spectral parameter. Moreover, the Lax representation of (10) is the spectral problem (18) and the auxiliary problem

$$\phi_t = V^{(2)}(u, v, \lambda)\phi \qquad V^{(2)}(u, v, \lambda) = \begin{pmatrix} V_{11}^{(2)} & V_{12}^{(2)} \\ V_{21}^{(2)} & -V_{11}^{(2)} \end{pmatrix} \tag{20}$$

where

$$\begin{aligned} V_{11}^{(2)} &= \tfrac{1}{2}\big(-\lambda^3 - 2\lambda v + u_{xx} + 3uu_x + u^3 + 2uv + 2v_x\big) \\ V_{12}^{(2)} &= -\lambda^2 v + \lambda(v_x - uv) - v_{xx} + u_x v + 2uv_x - u^2 v - 2v^2 \\ V_{21}^{(2)} &= \lambda^2 + \lambda u + u_x + u^2 + 2v. \end{aligned} \tag{21}$$

In order to construct a DT of (9) and (10), we first derive a gauge transformation for the spectral problem (18). Here we use the method in [20, 21, 24] to construct the gauge transformation. To this end, we choose $\varphi = (\varphi_1, \varphi_2)^{\mathrm{T}}$, $\psi = (\psi_1, \psi_2)^{\mathrm{T}}$ to be two basic solutions of the spectral problem (18) and use (φ, ψ) to define a 2×2 matrix T by

$$T = \begin{pmatrix} A_1(\lambda + A_0) & B \\ C & A_1^{-1}(\lambda + D_0) \end{pmatrix} \tag{22}$$

with

$$\begin{aligned} A_0 &= \frac{\lambda_1\alpha_2 - \lambda_2\alpha_1}{\alpha_1 - \alpha_2} & A_1^2 &= 1 + \frac{\alpha_1\alpha_2(\lambda_2 - \lambda_1)}{\alpha_1 - \alpha_2} \\ B &= \frac{A_1(\lambda_2 - \lambda_1)}{\alpha_1 - \alpha_2} & C &= A_1^{-1} - A_1 \\ D_0 &= \frac{\lambda_2\alpha_2 - \lambda_1\alpha_1}{\alpha_1 - \alpha_2} & \alpha_j &= \frac{\varphi_2(\lambda_j) - \gamma_j\psi_2(\lambda_j)}{\varphi_1(\lambda_j) - \gamma_j\psi_1(\lambda_j)} \end{aligned} \tag{23}$$

where parameters λ_j and γ_j ($j = 1, 2$; $\lambda_1 \neq \lambda_2$; $\gamma_1 \neq \gamma_2$) are suitably chosen such that all the denominators in (22) and (23) are not zero. From (22) and (23) we have

$$\det T = (\lambda - \lambda_1)(\lambda - \lambda_2). \tag{24}$$

We now introduce a gauge transformation

$$\hat{\phi} = T\phi \tag{25}$$

which transforms the spectral problem (18) and the auxiliary problems (19) and (20) into a spectral problem of $\hat{\phi}$

$$\hat{\phi}_x = \hat{U}\hat{\phi} \qquad \hat{U} = (T_x + TU)T^{-1} \tag{26}$$

and its auxiliary problems

$$\hat{\phi}_y = \hat{V}^{(1)}\hat{\phi} \qquad \hat{V}^{(1)} = \left(T_y + TV^{(1)}\right)T^{-1} \tag{27}$$

$$\hat{\phi}_t = \hat{V}^{(2)}\hat{\phi} \qquad \hat{V}^{(2)} = \left(T_t + TV^{(2)}\right)T^{-1}. \tag{28}$$

In a way similar to the proof in [24], we can verify the following fact.

Proposition 2. *The matrices $\hat{U}$, $\hat{V}^{(1)}$ and $\hat{V}^{(2)}$ determined by (26)–(28) have the same forms as U, $V^{(1)}$ and $V^{(2)}$, that is*

$$\hat{U} = U(\hat{u}, \hat{v}, \lambda) \qquad \hat{V}^{(1)} = V^{(1)}(\hat{u}, \hat{v}, \lambda) \qquad \hat{V}^{(2)} = V^{(2)}(\hat{u}, \hat{v}, \lambda) \tag{29}$$

where the transformation formulae from the old potentials u and v into new ones are given by

$$\hat{u} = u + \partial_x \ln A_1^2 \qquad \hat{v} = vA_1^2 - A_1 B. \tag{30}$$

According to proposition 2, it is easy to see that equations (26) and (27) are also a Lax pair of the (1 + 1)-dimensional soliton equation (9), and equations (26) and (28) are another Lax pair of the (1 + 1)-dimensional soliton equation (10). The transformation (30) is usually called a DT of the (1 + 1)-dimensional soliton equations (9) and (10). We obtain immediately the following assertion.

Proposition 3. *Let (u, v) be a solution of the $(1+1)$-dimensional soliton equations (9) and (10). Then (i) the function $(\hat{u}, \hat{v})$ determined by the DT (30) is a new solution of equations (9) and (10); (ii) the function*

$$\hat{q} = vA_1^2 - A_1 B \tag{31}$$

solves the KPII equation (1) and the $(2+1)$-dimensional equation (2), and the function

$$\hat{r} = u + \partial_x \ln A_1^2 \tag{32}$$

solves the $(2+1)$-dimensional equations (3)–(7).

In the following, we shall apply the DT to give explicit solutions of the (2+1)-dimensional evolution equations (1) and (7).

(i) Substituting the trivial solutions, $u = v = 0$, of (9) and (10) into (18)–(20), their two basic solutions φ, ψ are chosen as

$$\begin{aligned} \varphi &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{\frac{1}{2}\lambda x + \frac{1}{2}\lambda^2 y + \frac{1}{2}\lambda^3 t} \\ \psi &= \begin{pmatrix} -\lambda \\ 1 \end{pmatrix} e^{-\frac{1}{2}\lambda x - \frac{1}{2}\lambda^2 y - \frac{1}{2}\lambda^3 t}. \end{aligned} \tag{33}$$

Noticing equation (23), we have

$$\alpha_j = \frac{1}{\gamma_j \lambda_j} e^{\omega_j} - \frac{1}{\lambda_j} \qquad j = 1, 2 \tag{34}$$

$$\begin{aligned} A_1^2 &= 1 + \frac{(e^{\omega_1} - \gamma_1)(e^{\omega_2} - \gamma_2)}{a_2 e^{\omega_1} + a_1 e^{\omega_2} - \gamma_1 \gamma_2} \\ A_1 B &= \frac{A_1^2 \gamma_1 \gamma_2 \lambda_1 \lambda_2}{a_2 e^{\omega_1} + a_1 e^{\omega_2} - \gamma_1 \gamma_2} \end{aligned} \tag{35}$$

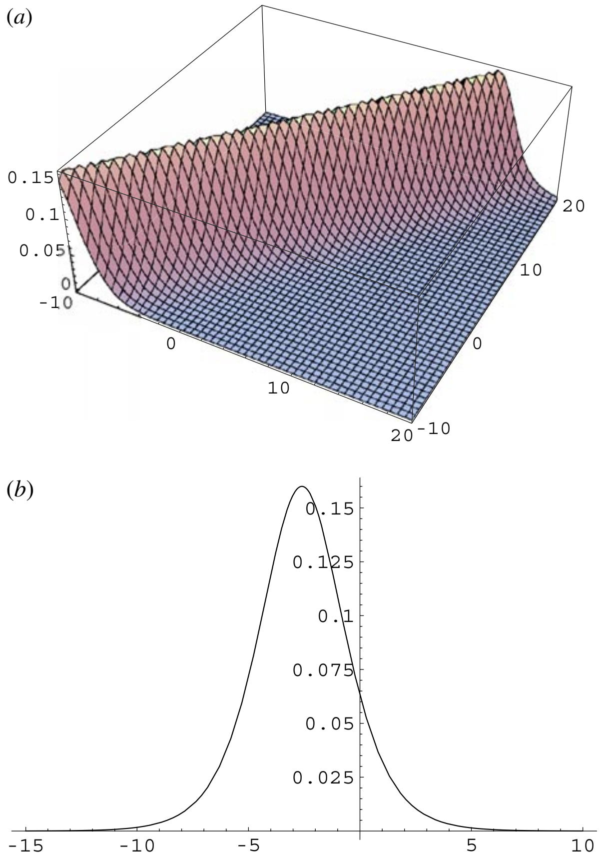


Figure 1. (*a*) $\hat{q}$ of $t = 0$ with $\gamma_1 = -1000$, $\gamma_2 = -0.01$, $\lambda = 0.001$, $\lambda_2 = -0.8$. (*b*) $\hat{q}$ of $y = 0$ and $t = 0$.

with

$$\omega_j = \lambda_j x + \lambda_j^2 y + \lambda_j^3 t \qquad j = 1, 2$$

$$a_1 = \frac{\gamma_1\lambda_1}{\lambda_1 - \lambda_2} \qquad a_2 = \frac{\gamma_2\lambda_2}{\lambda_2 - \lambda_1}.$$

Therefore, explicit solutions of the KPII equation (1) and the (2+1)-dimensional evolution equation (2) are obtained with the help of the DT (31) and (35):

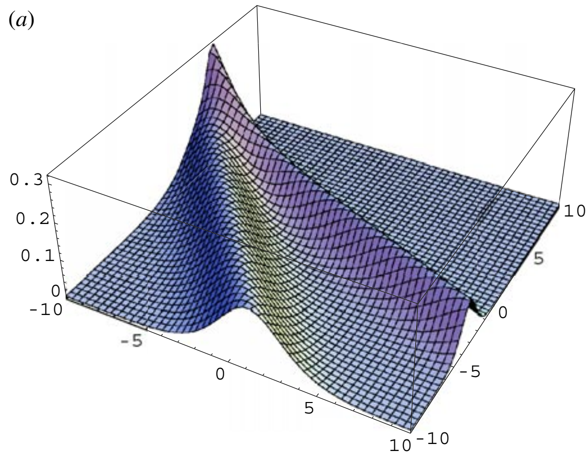
$$\hat{q} = -\gamma_1\gamma_2\lambda_1\lambda_2 \frac{e^{\omega_1+\omega_2} + (a_2 - \gamma_2)e^{\omega_1} + (a_1 - \gamma_1)e^{\omega_2}}{(a_2 e^{\omega_1} + a_1 e^{\omega_2} - \gamma_1\gamma_2)^2}. \tag{36}$$

It is easy to see that the function $\hat{q}$ has no singularity for $(\gamma_1, \gamma_2, \lambda_1, \lambda_2) \in \mathcal{A} \cup \mathcal{B}$, where $\mathcal{A}$ and $\mathcal{B}$ are defined by

$$\mathcal{A} = \{(\gamma_1, \gamma_2, \lambda_1, \lambda_2) \in \mathbb{R}^4 \mid \gamma_1 < 0, \gamma_2 < 0, \gamma_1 \neq \gamma_2 \text{ and } \lambda_1\lambda_2 < 0\}$$

$$\mathcal{B} = \{(\gamma_1, \gamma_2, \lambda_1, \lambda_2) \in \mathbb{R}^4 \mid \gamma_1 > 0, \gamma_2 < 0, \lambda_1 > \lambda_2 > 0 \text{ or } \gamma_1 < 0, \gamma_2 > 0, \lambda_2 < \lambda_1 < 0\}.$$

For given parameters $\gamma_1, \gamma_2, \lambda_1, \lambda_2$, (36) determines an exact solution of the (2 + 1)-dimensional evolution equations (1) and (2). If different parameters are chosen, the



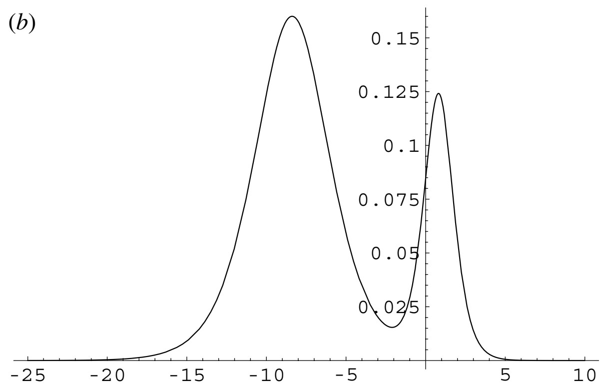


Figure 2. (*a*) $\hat{q}$ of $t = 0$ with $\gamma_1 = 0.02, \gamma_2 = -0.01, \lambda_1 = 1.5, \lambda_2 = 0.8$. (*b*) $\hat{q}$ of $x = 0$ and $t = 0$.

solution determined by (36) may be of different properties. For example, when $\gamma_1 = -1000, \gamma_2 = -0.01, \lambda_1 = 0.001, \lambda_2 = -0.8$, the function (36) is a one-soliton solution of the KPII equation (1) and the (2 + 1)-dimensional evolution equation (2) (see figures 1(*a*) and (*b*)), which is not same as those in [5, 6, 8–12, 14, 25] and [26]. When $\gamma_1 = 0.02, \gamma_2 = -0.01, \lambda_1 = 1.5, \lambda_2 = 0.8$, the function (36) gives a two-soliton solution of the KPII equation (1) and the (2 + 1)-dimensional evolution equation (2) (see figures 2(*a*) and (*b*)).

By using the DT (32), we arrive at explicit solutions of the mKP equation (3) and the (2 + 1)-dimensional evolution equations (4)–(7)

$$\hat{r} = \frac{(\lambda_1 + \lambda_2)e^{\omega_1+\omega_2} + \lambda_1(a_2 - \gamma_2)e^{\omega_1} + \lambda_2(a_1 - \gamma_1)e^{\omega_2}}{e^{\omega_1+\omega_2} + (a_2 - \gamma_2)e^{\omega_1} + (a_1 - \gamma_1)e^{\omega_2}} - \frac{\lambda_1 a_2 e^{\omega_1} + \lambda_2 a_1 e^{\omega_2}}{a_2 e^{\omega_1} + a_1 e^{\omega_2} - \gamma_1\gamma_2}. \tag{37}$$

Obviously, the function $\hat{r}$ has no singularity for $(\gamma_1, \gamma_2, \lambda_1, \lambda_2) \in \mathcal{A} \cup \mathcal{B}$. If the parameters $\gamma_1 = 100, \gamma_2 = -100, \lambda_1 = 1, \lambda_2 = 0.1$, the function (37) is a one-soliton solution of the mKP equation (3), which is not same as those in [27–31], and the (2 + 1)-dimensional evolution equations (4)–(7) (see figures 3(*a*) and (*b*)). For the parameters $\gamma_1 = 100, \gamma_2 = -0.1, \lambda_1 = 10, \lambda_2 = 9$, the function (37) describes a phenomenon (see figures 4(*a*) and (*b*)). Here a general method to choose the parameters is to guarantee that

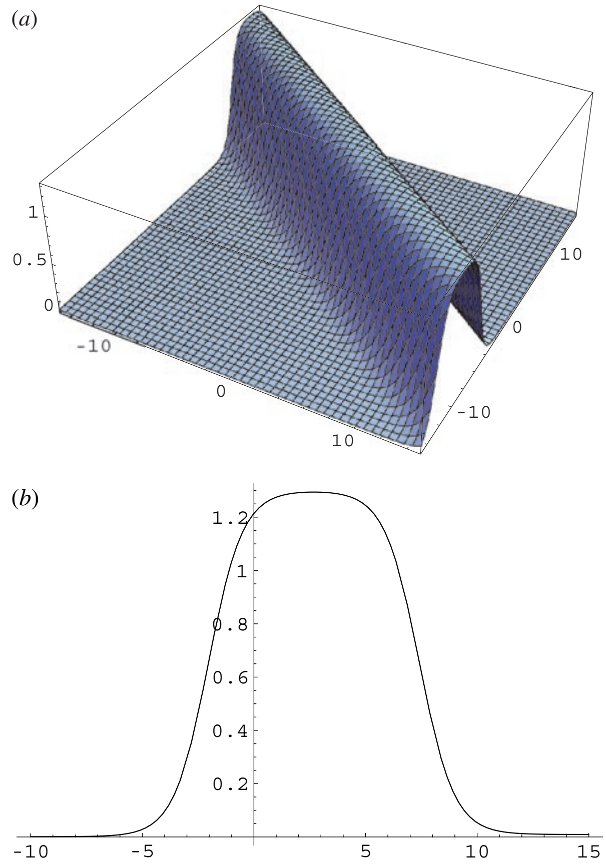


Figure 3. (*a*) $\hat{r}$ of $t = 0$ with $\gamma_1 = 10$, $\gamma_2 = -0.1$, $\lambda_1 = 1.3$, $\lambda_2 = 0.01$. (*b*) $\hat{r}$ of $y = 0$ and $t = 0$.

the denominator in (36) or (37) is not zero, that is, $(\gamma_1, \gamma_2, \lambda_1, \lambda_2) \in \mathcal{A} \cup \mathcal{B}$. Then the properties of solutions are compared for various choices of parameters.

(ii) It is obvious that $u = 0$ and $v = -1$ are also solutions of (9) and (10). Substituting into (18)–(20) yields two basic solutions

$$\varphi = \begin{pmatrix} -\frac{1}{2}\left(\lambda - \sqrt{\lambda^2+4}\right) \\ 1 \end{pmatrix} \mathrm{e}^{\frac{1}{2}\sqrt{\lambda^2+4}[x+\lambda y+(\lambda^2-2)t]}$$
$$\psi = \begin{pmatrix} -\frac{1}{2}\left(\lambda + \sqrt{\lambda^2+4}\right) \\ 1 \end{pmatrix} \mathrm{e}^{-\frac{1}{2}\sqrt{\lambda^2+4}[x+\lambda y+(\lambda^2-2)t]}. \tag{38}$$

Using equation (23), we have

$$\alpha_j = \frac{\mathrm{e}^{w_j} - \gamma_j}{\mu_j \mathrm{e}^{w_j} + \kappa_j \gamma_j} \qquad j = 1, 2 \tag{39}$$

$$A_1^2 = 1 + \frac{(\mathrm{e}^{w_1} - \gamma_1)(\mathrm{e}^{w_2} - \gamma_2)}{b_1 \mathrm{e}^{w_1+w_2} + b_2 \mathrm{e}^{w_1} + b_3 \mathrm{e}^{w_2} + b_4}$$
$$A_1 B = \frac{A_1^2 (\mu_1 \mathrm{e}^{w_1} + \kappa_1 \gamma_1)(\mu_2 \mathrm{e}^{w_2} + \kappa_2 \gamma_2)}{b_1 \mathrm{e}^{w_1+w_2} + b_2 \mathrm{e}^{w_1} + b_3 \mathrm{e}^{w_2} + b_4} \tag{40}$$

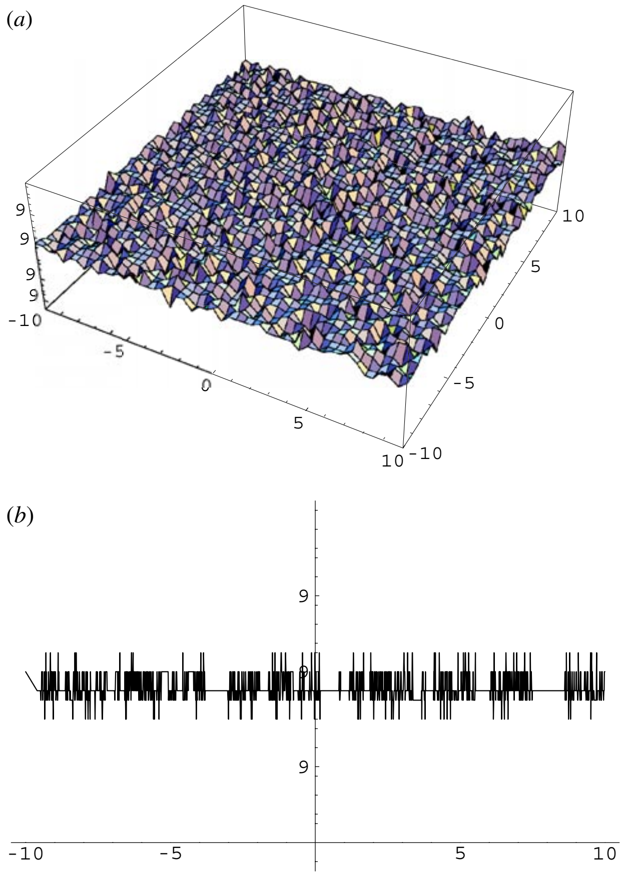


Figure 4. (*a*) $\hat{r}$ of $t = 4$ with $\gamma_1 = 100$, $\gamma_2 = -0.1$, $\lambda_1 = 10$, $\lambda_2 = 9$. (*b*) $\hat{r}$ of $x = 0$ and $t = 4$.

with

$$\begin{aligned}
&w_j = \nu_j[x + \lambda_j y + (\lambda_j^2 - 2)t] \qquad \nu_j = \sqrt{\lambda_j^2 + 4}\\
&\mu_j = \tfrac{1}{2}(\nu_j - \lambda_j) \qquad \kappa_j = \tfrac{1}{2}(\nu_j + \lambda_j) \qquad j = 1, 2,\\
&b_1 = \frac{\mu_1 - \mu_2}{\lambda_1 - \lambda_2} \qquad b_2 = -\frac{\gamma_2(\mu_1 + \kappa_2)}{\lambda_1 - \lambda_2}\\
&b_3 = \frac{\gamma_1(\mu_2 + \kappa_1)}{\lambda_1 - \lambda_2} \qquad b_4 = -\frac{\gamma_1\gamma_2(\kappa_1 - \kappa_2)}{\lambda_1 - \lambda_2}.
\end{aligned} \tag{41}$$

Thus, using the DT (31), equations (40) and (41), we obtain explicit solutions of the KPII equation (1) and the (2 + 1)-dimensional equation (2)

$$\begin{aligned}
\hat{q} = -1 &- \frac{(\mu_1 e^{w_1} + \kappa_1\gamma_1)(\mu_2 e^{w_2} + \kappa_2\gamma_2) + (e^{w_1} - \gamma_1)(e^{w_2} - \gamma_2)}{b_1 e^{w_1+w_2} + b_2 e^{w_1} + b_3 e^{w_2} + b_4}\\
&- \frac{(\mu_1 e^{w_1} + \kappa_1\gamma_1)(\mu_2 e^{w_2} + \kappa_2\gamma_2)(e^{w_1} - \gamma_1)(e^{w_2} - \gamma_2)}{(b_1 e^{w_1+w_2} + b_2 e^{w_1} + b_3 e^{w_2} + b_4)^2}.
\end{aligned} \tag{42}$$

Resorting to the DT (32) and (40), we have

$$\begin{aligned}
\hat{r} = &\frac{(\nu_1 + \nu_2)(1 + b_1)e^{w_1+w_2} + \nu_1(b_2 - \gamma_2)e^{w_1} + \nu_2(b_3 - \gamma_1)e^{w_2}}{(1 + b_1)e^{w_1+w_2} + (b_2 - \gamma_2)e^{w_1} + (b_3 - \gamma_1)e^{w_2} + b_4 + \gamma_1\gamma_2}\\
&- \frac{b_1(\nu_1 + \nu_2)e^{w_1+w_2} + \nu_1 b_2 e^{w_1} + \nu_2 b_3 e^{w_2}}{b_1 e^{w_1+w_2} + b_2 e^{w_1} + b_3 e^{w_2} + b_4}
\end{aligned} \tag{43}$$

which are also explicit solutions of the mKP equation (3) and the (2 + 1)-dimensional evolution equations (4)–(7).

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